CHAPTER 6

WORK AND ENERGY

OBJECTIVES

After studying the material of this chapter, the student should be able to:

- distinguish between the scientific definition of work as compared to the colloquial definition.
- write the definition of work in terms of force and displacement and calculate the work done by a constant force when the force and displacement vectors are at an angle.
- use graphical analysis to calculate the work done by a force that varies in magnitude.
- define types of mechanical energy and give examples of types of energy that are not mechanical.
- state the work-energy theorem and apply the theorem to solve problems.
- distinguish between a conservative and a nonconservative force and give examples of each type of force.
- state the law of conservation of energy and apply the law to problems involving mechanical energy.
- define power in the scientific sense and solve problems involving work and power.

KEY TERMS AND PHRASES

work is done on an object when the energy of the object changes. The work done equals the product of the net force (F), the displacement (d), and the cosine of the angle between the force vector and the displacement vector.

joule (J) is the unit associated with work and energy. $1\ J = 1\ N\ m = 1\ kg\ m/s^2$.

kinetic energy (KE) is energy due to an object's motion. Translational kinetic energy of an object depends on the object's mass and the square of its speed.

work-energy theorem states that the net work done on an object is equal to its change in kinetic energy.

gravitational potential energy (PE) is energy stored by an object due to its position. Near the surface of the Earth, the gravitational potential energy relative to some reference point, e.g., the ground, is given by the product of the object's weight and the height above the reference level.

elastic potential energy is energy that results from an object being stretched or compressed (e.g., a spring), or twisted as in case of a wire. The change in a spring's elastic potential energy as it is stretched (or compressed) is related to the force (F) required to stretch (or compress) the spring and the distance that the spring is stretched (or compressed).

conservative force does the same amount of work independent of the path taken between two points. An example of a conservative force is gravity. The work done equals the change in potential energy ($W = \Delta PE = mg\ \Delta y$) and depends only on the initial and final positions above the ground and not on the path taken.

nonconservative force results in different amounts of work being expended in moving an object. The net work required depends on the path taken moving the object between points. For example, the work done in sliding a box of books against friction from one end of a room to the other depends on the path taken.

mechanical energy refers to an object's kinetic energy and potential energy.

law of conservation of energy states that energy is neither created nor destroyed. Energy can be transformed from one kind to another, but the total amount remains constant.

power is the rate at which work is done and is measured in watts where 1 watt = 1 J/s.

SUMMARY OF MATHEMATICAL FORMULAS

work	$W = F\,d \cos\theta$	The work done depends on the net force, displacement, and the angle between the force vector and the displacement vector.
kinetic energy	$KE = \frac{1}{2} m v^2$	Kinetic energy is energy of motion. Kinetic energy depends on an object's mass and the square of its speed.
work-energy theorem	$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$	The work-energy theorem states that the net work done on an object is equal to its change in kinetic energy.
gravitational potential energy	$PE = m\,g\,y$	For an object near the surface of the Earth, the gravitational potential energy depends on its weight and its height above a reference level.
spring or elastic potential energy	$PE = \frac{1}{2} k x^2$	Elastic potential energy depends on the force constant of the spring (k) and the displacement from equilibrium position (x).
power	$P = W/t$ or $P = F\,\bar{v} \cos\theta$	Power is the rate at which work is done, or for an object traveling at constant speed, the power expended is related to the net force and the average speed.

CONCEPT SUMMARY

Work and Energy

The **work** (W) done by a force (F) that is constant in both magnitude and direction is given by the following equation:

$W = F\ d \cos \theta$

where d is the displacement of the object and θ is the angle between the force vector and the displacement vector.

Work is a scalar quantity. Since work is the product of a force and displacement, the unit of work is newton meter. The newton meter has been named the **joule** (J), and 1 J = 1 N m.

If the force acting on an object varies in magnitude and/or direction during the object's displacement, graphical analysis can be used to determine the work done. $F \cos \theta$ is plotted on the y axis and the distance through which the object moves is plotted on the x axis. The work done is represented by the area under the curve. The sum of the complete and partial blocks under the curve is determined and the work done equals the product of the work represented by one block and the total number of blocks.

A moving object is said to have **kinetic energy** (KE). The translational kinetic energy of an object depends on the object's mass and the square of its speed. The formula for kinetic energy is

$KE = \frac{1}{2}\ m\ v^2$

When a force does work on an object, the energy of the object will be changed by an amount equal to the amount of work done. The work done is positive if the energy of the object increases, for example, a car accelerating from rest. The work done is negative if the energy of the object decreases; for example, the kinetic energy of a sliding object on a rough surface is gradually lost as the object comes to a halt. In this case the kinetic energy is dissipated into the form of heat energy. The **work-energy theorem** states that the net work done on an object is equal to its change in kinetic energy, i.e., $W = \Delta KE = \frac{1}{2}\ m\ v_2^2 - \frac{1}{2}\ m\ v_1^2$.

Gravitational potential energy (PE) is energy stored by an object due to its position. The formula for the gravitational potential energy of an object relative to some reference point, e.g., the ground, is given by

$PE = m\ g\ y$ where y = the difference in height and the reference level

If an object of mass m is raised from an initial height y_1 and raised to a height y_2 above its original position, the change in gravitational potential energy is given by

$\Delta PE = m\ g\ y_2 - m\ g\ y_1$

Elastic potential energy is energy that results from an object being stretched or compressed (e.g., a spring), or twisted as in case of a wire. The force (F) required to stretch or compress a spring is given by $F = k\ x$ where k is the force constant measured in newtons/meter (N/m) and x is the stretch or compression. The change in a spring's elastic potential energy as it is stretched or compressed is given by

$\Delta PE = \frac{1}{2} k x_2^2 - \frac{1}{2} k x_1^2$ where x_1 = the initial displacement from the equilibrium
and x_2 = the final displacement from equilibrium

TEXTBOOK QUESTION 2. Can a centripetal force ever do work on an object? Explain.

ANSWER: For an object in traveling in *circular* motion, the centripetal force *never* does work. The centripetal force is directed toward the center of the circle while the displacement vector is tangent to the circle. Therefore, the centripetal force vector is always perpendicular to the displacement vector ($\theta = 90°$). However, $\cos 90° = 0$; therefore, $W = F\,d \cos 90° = 0$.

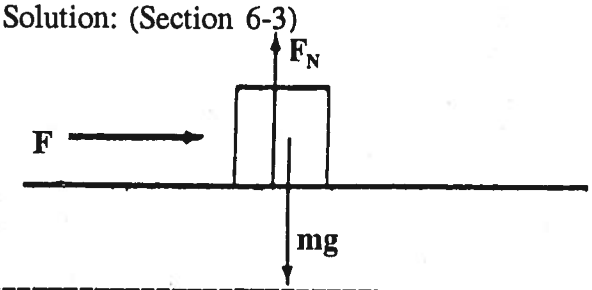
An alternative answer can be given from the work-energy theorem. The centripetal force changes the direction of motion of an object traveling in a circle without changing the object's speed. If the object's speed does not change, then its kinetic energy does not change and no work was done on the object. Note: if the motion is elliptical instead of circular, then θ is not equal to 90° and work is done on the object.

EXAMPLE PROBLEM 1. A student exerts a horizontal force of 20.0 N with her hand and pushes a 10.0 kg box a distance of 2.00 m across a frictionless floor. Calculate the a) magnitude of the work done by the student and b) velocity of the box at the 2.00 m mark.

Part a. Step 1.

Draw an accurate diagram locating all of the forces acting on the box.

Solution: (Section 6-3)



Part a. Step 2.

Calculate the work done. Note: the angle between the force and the displacement vector is 0°.

$W = F\,d \cos\theta = (20.0\text{ N})(2.00\text{ m}) \cos 0° = 40.0\text{ J}$

Note: the weight of the box is balanced by the normal force exerted by the floor.

Part a. Step 3.

Use the work-energy theorem to solve for the final velocity.

$W = \Delta KE = \frac{1}{2} m v'^2 - \frac{1}{2} m v^2$

$40.0\text{ J} = \frac{1}{2}(10.0\text{ kg}) v'^2 - \frac{1}{2}(10.0\text{ kg})(0\text{ m/s})^2$

$v'^2 = 2(40.0\text{ J})/(10.0\text{ kg}) = 8.00\text{ m}^2/\text{s}^2$

$v' = 2.83\text{ m/s}$

Conservative and Nonconservative Forces

The work done by a **conservative force** depends only on the initial and final position of the object acted upon. An example of a conservative force is gravity. The work done equals the change in potential energy ($W = \Delta PE = mg\ \Delta y$) and depends only on the initial and final positions above the ground and *not* on the path taken.

Friction is a **nonconservative force** and the work done in moving an object against a nonconservative force depends on the path. For example, the work done in sliding a box of books against friction from one end of a room to the other depends on the path taken.

Law of Conservation of Energy

The **law of conservation of energy** states that energy is neither created nor destroyed. Energy can be transformed from one kind to another, but the total amount remains constant. For mechanical systems involving conservative forces, the total **mechanical energy** equals the sum of the kinetic and potential energies of the objects that make up the system.

$E = KE + PE$

An example of the law of conservation of energy involving a mechanical system is a ball thrown vertically upward. Neglecting air resistance, kinetic energy is gradually transformed into potential energy and then back into kinetic energy. At each point of the motion the sum of the kinetic energy and potential energy remains constant.

TEXTBOOK QUESTION 10. In Fig. 6-31, water balloons are tossed from the roof of a building, all with the same speed but with different launch angles. Which one has the highest speed on impact? Ignore air resistance.

ANSWER: When the water balloon is initially released it has both potential and kinetic energy. Assuming that air resistance is negligible and that the surface of the ground below is the zero of potential energy, the balloon's energy is all kinetic energy when it is about to strike the ground.

$PE_{building} + KE_{building} = KE_{ground}$

Each balloon's kinetic energy as it strikes the ground depends only on the total initial energy and not the angle at which it was thrown. The balloon's speed is a scalar quantity and at the ground (v_{ground}) is related to the kinetic energy KE_{ground} by the equation $KE = \frac{1}{2} m v^2$. Therefore, the balloon's speed as it is about to strike the ground is independent of the launch angle.

If air resistance is NOT negligible, then the speed does depend on the launch angle. A balloon thrown upward at some angle to the horizontal will encounter more air resistance because it travels a greater distance through the air than a balloon thrown vertically downward. Because of this, if air resistance is a factor, a balloon thrown vertically downward will have the greatest speed.

TEXTBOOK QUESTION 13. A bowling ball is hung from the ceiling by a steel wire (Fig. 6-33). The instructor pulls the ball back and stands against the wall with the ball against his nose. To avoid injury the instructor is suppose to release the ball without pushing it. Why?

ANSWER: At release, the ball has potential energy (PE) but no kinetic energy (KE). Based on the law of conservation of energy, the total energy cannot exceed the initial energy. Therefore, as the ball swings away from the instructor, the total KE + PE equals the initial PE. Assuming frictional losses are negligible, when the ball returns it will barely touch the instructor's nose before moving away again.

However, if the ball is pushed it will have both PE + KE and when it returns it will tend to reach a higher point above the ground than the point where the instructor has placed his nose. The lecturer will be hit by a moving bowling ball.

Note: as a young teacher, the author of this study guide performed this particular experiment using a student volunteer. A 2 kg cylinder attached to the ceiling by a thick string was displaced and released. Unfortunately, the string broke as the object was returning to a student's nose and the cylinder struck the student in the stomach. Since that experience, the demonstration has been performed as a "thought" experiment.

TEXTBOOK QUESTION 14. What happens to the gravitational potential energy when water at the top of a waterfall falls to the pool below?

ANSWER: If air resistance is negligible, then the gravitational potential energy and kinetic energy of the water at the top of the waterfall is converted into kinetic energy at the pool below. When the falling water strikes the pool below then it is converted into thermal energy. Therefore, the water in the pool below should be at a higher temperature than the water at the top of the waterfall.

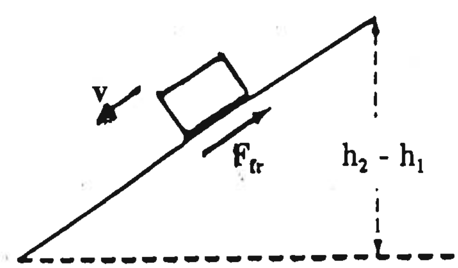
James Joule, a British physicist, measured the temperature of the water at both the top and bottom of a high waterfall and found that the temperature at both locations was about the same. He theorized that evaporative cooling of the falling water as it mixed with the air led to essentially no change in temperature.

EXAMPLE PROBLEM 2. A stone is thrown vertically upward with an initial speed of 24.5 m/s from the edge of the roof of a building 29.4 m high. The stone just misses the edge of the building on the way down and strikes the street below. Use the law of conservation of energy to determine the a) maximum height above the ground reached by the stone and b) stone's velocity just before it strikes the ground.

Part a. Step 1.	Solution: (Sections 6-6 and 6-7)
Complete a data table.	Let point 1 represent the edge of the roof, point 2 represent maximum height, and point 3 just above ground level.
	$h_1 = 29.4$ m $\quad v_1 = 24.5$ m/s $\qquad g = 9.80$ m/s^2

	h_2 = ? v_2 = 0 m/s h_3 = 0 m v_3 = ?
Part a. Step 2. Use the law of conservation of energy to determine the maximum height reached by the stone.	$PE_1 + KE_1 = PE_2 + KE_2$ $mgh_1 + ½\, m\, v_1^2 = mgh_2 + ½\, m\, v_2^2$ Note: the mass cancels. $(9.80\ m/s^2)(29.4\ m) + ½\ (24.5\ m/s)^2 = (9.80\ m/s^2)\ h_2 + ½\ (0\ m/s)^2$ $288\ m^2/s^2 + 300\ m^2/s^2 = (9.80\ m/s^2)\ h_2$ $h_2 = 60.0\ m$
Part b. Step 1 Use the law of conservation of energy to determine the stone's speed just before it struck the ground.	$PE_2 + KE_2 = PE_3 + KE_3$ $mgh_2 + ½\, m\, v_2^2 = mgh_3 + ½\, m\, v_3^2$ Note: the mass cancels. $(9.80\ m/s^2)(60.0\ m) + ½\ (0\ m/s)^2 = (9.80\ m/s^2)(0\ m) + ½\ v_3^2$ $588\ m^2/s^2 = ½\ v_3^2$ $v_3 = 34.2\ m/s$

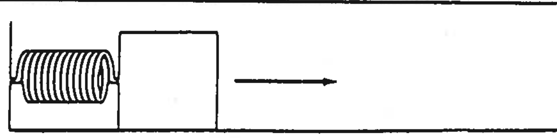
EXAMPLE PROBLEM 3. A 5.00 kg box is pushed from the top of an incline and travels down the incline with an initial speed of 10.0 m/s. The incline is 5.00 m long and the angle of the incline is 30.0°. The coefficient of friction between the box and the incline is 0.200. Determine the a) loss of potential energy and b) work done by friction. c) Use the work-energy principle to determine the velocity of the box at the bottom of the incline.



Part a. Step 1. Complete a data table.	Solution: (Sections 6-1, 6-4, and 6-9) $h_1 = (5.00\ m)(\sin 30.0°) = 2.50\ m$ $h_2 = 0\ m$ $v_1 = 10.0\ m/s$ v_2 = ? $F_{fr} = \mu\, mg \cos\theta = (0.200)(5.00\ kg)(9.80\ m/s^2)(\cos 30.0°) = 8.49\ N$
Part a. Step 2. Determine the loss of potential energy.	$\Delta PE = mgh_2 - mgh_1 = mg(h_2 - h_1)$ $= (5.00\ kg)(9.80\ m/s^2)(0\ m - 2.50\ m)$ $\Delta PE = -\ 123\ J$

Part b. Step 1. Determine the work done by friction.	The frictional force and the direction of motion are in opposite directions; therefore, $\theta = 180°$. $W = F\,d\cos\theta = (8.49\ \text{N})(5.00\ \text{m})(\cos 180°) = -42.5\ \text{J}$ The mechanical energy lost by the system equals the work done by friction.
Part c. Step 1. Determine the total mechanical energy at the top of the incline.	The total energy at the top of the slide is the sum of the PE and KE at that point. total energy $= (5.00\ \text{kg})(9.80\ \text{m/s}^2)(2.50\ \text{m}) + \frac{1}{2}(5.00\ \text{kg})(10.0\ \text{m/s})^2$ total energy $= 123\ \text{J} + 250\ \text{J} = 373\ \text{J}$
Part c. Step 2. Use the law of conservation of energy to solve for the speed of the box at the bottom of the incline.	The total kinetic + potential energy at the top of the final section is greater than that at the bottom by an amount equal to the mechanical energy lost due to friction. $PE_1 + KE_1 +$ energy lost(friction) $= PE_2 + KE_2$ $373\ \text{J} + -42.5\ \text{J} = (5.00\ \text{kg})(9.80\ \text{m/s}^2)(0\ \text{m}) + \frac{1}{2}(5.00\ \text{kg})\,v_2^2$ $331\ \text{J} = (2.50\ \text{kg})\,v_2^2$ $v_2^2 = 132\ \text{m}^2/\text{s}^2$ $v_2 = 11.5\ \text{m/s}$

EXAMPLE PROBLEM 3. As shown in the diagram, a 1.00 kg wooden block is on a level board and held against a spring of force constant 30.0 N/m which has been compressed 0.200 m. The block is released and propelled horizontally across the board. The coefficient of friction between the block and the board is 0.20. Determine the a) velocity of the block just as it leaves the spring and b) distance that the block travels after it leaves the spring. Assume that friction between the block and board is negligible until the point where the block leaves the spring.



Part a. Step 1. Use the law of conservation of energy to determine the block's speed as it leaves the spring.	Solution: (Sections 6-7 and 6-8) $\Delta PE_{spring} = \Delta KE_{block}$ $\frac{1}{2}k\,x^2 = \frac{1}{2}m\,v^2$ and rearranging gives $v = (k/m)^{1/2}\,x = [(30.0\ \text{N/m})/(1.00\ \text{kg})]^{1/2}\,(0.200\ \text{m})$ $v = 1.10\ \text{m/s}$

Part b. Step 1.

Use the work-energy theorem to determine the distance the block slides.

work = change in kinetic energy

$F\ d \cos \theta = \frac{1}{2} m v'^2 - \frac{1}{2} m v^2$

The force acting on the block is due to friction.

$F = F_{fr} = \mu_k F_N$, where $F_N = mg$.

The frictional force is directed opposite from the direction of motion; therefore, $\cos \theta = \cos 180° = -1$.

The block comes to rest; therefore, the final velocity of the block is zero ($v' = 0$).

$\mu_k mg\ d \cos 180° = \frac{1}{2} m v'^2 - \frac{1}{2} m v^2$ The mass cancels; therefore,

$(0.200)(9.80\ m/s^2)\ d\ (-1) = 0 - \frac{1}{2}(1.10\ m/s)^2$

$d = 0.309$ m

Power

Power is defined as the rate at which work is done. The average power can be determined by applying the following formula:

power = work/time = energy transformed/time.

$P = W/t$

The unit of power is the **watt,** where 1 watt = 1 joule/second. The watt is related to the English unit of horsepower: 746 watts = 1 horsepower. The power output of a constant force (F) applied to an object is given by

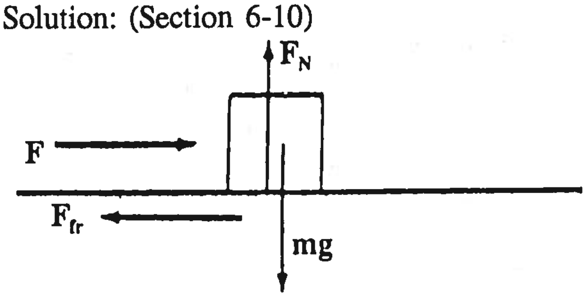
$P = F\ \bar{v}$ where $\bar{v} = d/t$ is the average speed of the object during the time interval being considered.

EXAMPLE PROBLEM 4. A man pushes a 50.0 kg box across a level floor at a constant speed of 1.00 m/s for 10.0 s. If the coefficient of friction between the box and the floor is 0.200, determine the average power output by the man.

Part a. Step 1.

Draw an accurate diagram locating all of the forces acting on the box.

Solution: (Section 6-10)



Part a. Step 2. Complete a data table.	$m = 50.0$ kg $\quad \mu_k = 0.200$ $\bar{v} = 1.00$ m/s $\quad t = 10$ s
Part a. Step 3. Apply Newton's second law of motion and determine the force (F) exerted by the man.	$\Sigma F = m\,a$ but $a = 0$ m/s^2 (speed is constant) $F - F_{fr} = m\,(0 \text{ m/s}^2)$ but $F_{fr} = \mu_k F_N$ $F - \mu_k F_N = 0$ and $F_N = mg$ $F - \mu_k m g = 0$ $F - (0.20)(50.0 \text{ kg})(9.80 \text{ m/s}^2) = 0$ $F = 98.0$ N
Part a. Step 4. Determine the average power output of the man.	$P = W/t = F\,d/t \cos 0^\circ$ The speed is constant; therefore, $d/t = v$ $P = F\,v \cos 0^\circ = (98.0 \text{ N})(1.0 \text{ m/s})$ $P = 98.0$ W

PROBLEM SOLVING SKILLS

For problems involving the work-energy theorem:

1. Draw an accurate diagram locating all of the forces, both conservative and nonconservative, acting on the object.
2. Determine the magnitude and direction of the net force acting on the object and then determine the net work done on the object.
3. Apply the work-energy theorem and solve the problem.

For problems involving the law of conservation of energy:

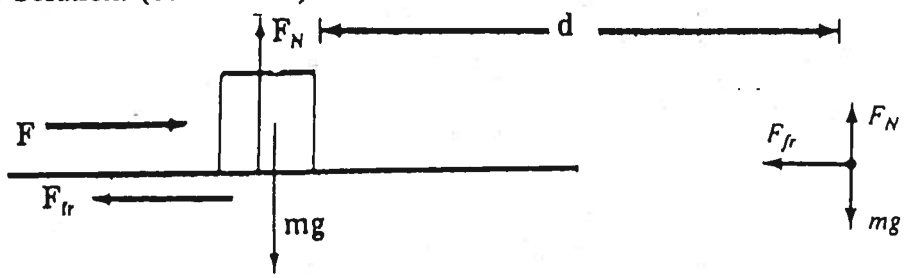
1. Draw an accurate diagram locating all of the forces, both conservative and nonconservative, acting on the object.
2. Apply the law of conservation of energy and solve the problem.

For problems involving power:

1. Draw an accurate diagram locating all of the forces, both conservative and nonconservative, acting on the object.
2. Apply the formulas for power and solve the problem.

SOLUTIONS TO SELECTED TEXTBOOK PROBLEMS

TEXTBOOK PROBLEM 4. How much work did the movers do (horizontally) pushing a 160 kg crate 10.3 m across a rough floor without acceleration, if the effective coefficient of friction was 0.50?

Part a. Step 1. Draw an accurate diagram locating all of the forces acting on the crate.	Solution: (Section 6-1) 
Part a. Step 2. Complete a data table.	m = 160 kg g = 9.8 m/s² μ = 0.50 θ = 0° d = 10.3 m
Part a. Step 3. Determine the magnitude of the force F.	$\Sigma F = m\,a$ but $a = 0\ m/s^2$ $F - F_{fr} = m\,(0\ m/s^2)$ therefore, $F = F_{fr}$ From textbook, chapter 4, $F_{fr} = \mu\,F_N = \mu\,m\,g\cos\theta$ $F_{fr} = (0.50)(160\ kg)(9.8\ m/s^2)\cos 180° = -784\ N$
Part a. Step 4. Determine the work done.	$W = F\,d\cos\theta = (784\ N)(10.3\ m)\cos 0°$ $W = 8.1 \times 10^3\ J$

TEXTBOOK PROBLEM 20. A baseball (m = 140 grams) traveling 32 m/s moves a fielder's glove backward 25 cm when the ball is caught. What is the average force exerted on the ball by the glove?

Part a. Step 1. Complete a data table.	Solution: (Section 6-3) m = 140 grams = 0.140 kg v_o = 32 m/s v = 0 m/s F = ? d = 25 cm = 0.25 m θ = 180°
Part a. Step 2. Apply the work-energy theorem and solve for the distance.	$W = \Delta KE$ $F\,d\cos\theta = \frac{1}{2}\,m\,v'^2 - \frac{1}{2}\,m\,v^2$ $F\,(0.25\ m)(\cos 180°) = \frac{1}{2}(0.140\ kg)(0\ m/s)^2 - \frac{1}{2}(0.140\ kg)(32.0\ m/s)^2$ $-F\,(0.25\ m) = 0\ J - 71.7\ J$ $F = 2.9 \times 10^2\ N$ Note: the value of θ is 180° because the force the glove exerts is directed opposite from the direction of the ball's motion.

TEXTBOOK PROBLEM 22. At an accident scene on a level road, investigators measure a car's skid mark to be 88 m long. The accident occurred on a rainy day and the coefficient of kinetic friction was estimated to be 0.42. Use these data to determine the speed of the car when the driver slammed on (and locked) the brakes. (Why does the mass not matter?)

Part a. Step 1.

Use the work-energy theorem to determine the car's initial speed.

Solution: (Section 6-3)

$W = F\, d \cos\theta = \Delta KE$ where $F = -F_{fr} = \mu_s\, m\, g \cos 180°$

$\mu_s\, m\, g \cos 180°\, d = \frac{1}{2} m v'^2 - \frac{1}{2} m v^2$ Note: mass m cancels

$(0.42)(9.8\ m/s^2)(-1.0)(88\ m) = \frac{1}{2}(0\ m/s)^2 - \frac{1}{2} v^2$

$v^2 = 720\ m^2/s^2$

$v = 27\ m/s \approx 60$ miles per hour

Note: the value of θ is 180° because the braking force acts in the opposite direction from the direction of the car's motion.

TEXTBOOK PROBLEM 37. A 65 kg trampoline artist jumps vertically upward from the top of a platform with a speed of 5.0 m/s. (a) How fast is he going as he lands on the trampoline, 3.0 m below (Fig. 6-38)? (b) If the trampoline behaves like a spring of spring constant 6.2×10^4 N/m, how far does he depress it?

Part a. Step 1.

Complete a data table. Measure all distances from the point where the artist first touches the trampoline.

Solution: (Sections 6-6 and 6-7)

Let point 1 represent the top of the platform and point 2 represent the top of the trampoline.

$g = 9.8\ m/s^2$ $h_1 = 3.0\ m$ $v_1 = 5.0\ m/s$

$h_2 = 0$ $v_2 = ?\ m/s$

Part a. Step 2.

Use the law of conservation of energy to determine the artist's speed as he strikes the trampoline.

$PE_1 + KE_1 = PE_2 + KE_2$

$mgh_1 + \frac{1}{2} m v_1^2 = mgh_2 + \frac{1}{2} m v_2^2$ Note: the mass cancels

$(9.8\ m/s^2)(3.0\ m) + \frac{1}{2}(5.0\ m/s)^2 = (9.8\ m/s^2)(0) + \frac{1}{2} v_2^2$

$29.4\ m^2/s^2 + 12.5\ m^2/s^2 = \frac{1}{2} v_2^2$

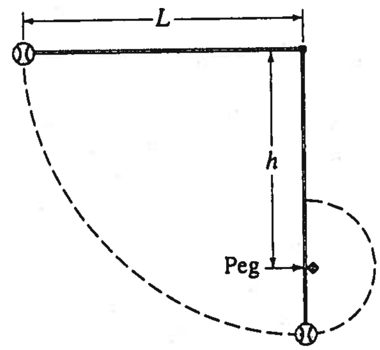
$v_2 = 9.2\ m/s$

Part b. Step 1. Complete a data table using data for points 2 and 3.	Let point 3 represent the maximum depression of the trampoline. This point is below the top of the trampoline. Therefore, let the maximum depression be represented by Y and $h_3 = -Y$. $g = 9.8\ m/s^2$ $v_2 = 9.2\ m/s$ $h_2 = 0\ m$ $h_3 = -Y$ $v_3 = 0\ m/s$
Part b. Step 2. Use the law of conservation of energy to determine the maximum depression of the trampoline.	$PE_2 + KE_2 = PE_{3(gravity)} + KE_3 + PE_{3(trampoline)}$ where $h_3 = -Y$, and $v_3 = 0\ m/s$, then $KE_3 = 0\ J$ $mgh_2 + \frac{1}{2}\, m\, v_2^2 = mgh_3 + \frac{1}{2}\, m\, v_3^2 + \frac{1}{2}\, k\, y_3^2$ $(65.0\ kg)(9.80\ m/s^2)(0\ m) + \frac{1}{2}\,(65.0\ kg)\,(9.2\ m/s)^2 =$ $(65.0\ kg)(9.80\ m/s^2)(-Y) + 0\ J + \frac{1}{2}\,(6.2 \times 10^4\ N/m)\ Y^2$ $0\ J + 2750\ J = (637\ N)(-Y) + (3.1 \times 10^4)\ Y^2$ rearranging gives $0 = (3.1 \times 10^4)\ Y^2 - 637\ Y - 2750$ Using the quadratic formula to determine the value of Y gives $Y = 0.31\ m$ or $Y = -0.29\ m$. The positive value of Y represents the maximum depression; therefore, the correct answer is $Y = 0.31\ m$.

TEXTBOOK PROBLEM 65. A shot-putter accelerates a 7.3 kg shot from rest to 14 m/s. If this motion takes 1.5 s, what average power was developed?

Part a. Step 1. Determine the change in the object's kinetic energy.	Solution: (Section 6-10) $\Delta KE = KE_f - KE_i$ $= \frac{1}{2}\, m\, v_f^2 - \frac{1}{2}\, m\, v_i^2$ $= \frac{1}{2}\,(7.3\ kg)(14\ m/s)^2 - \frac{1}{2}\,(7.3\ kg)(0\ m/s)^2$ $\Delta KE = 720\ J$
Part a. Step 2. Determine the average power expended by the shot-putter.	$P = W/t = \Delta KE/t$ $= (720\ J)/(1.5\ s)$ $P = 4.8 \times 10^2\ W$

TEXTBOOK PROBLEM 77. A ball is attached to a horizontal cord of length L whose other end is fixed (Fig 6-43). (a) If the ball is released, what will be its speed at the lowest point in its path? (b) A peg is located a distance h directly below the point of attachment of the cord. If h = 0.80 L, what will be the speed of the ball when it reaches the top of its circular path about the peg?



Part a. Step 1. Use the law of conservation of energy to solve for the ball's speed at the lowest point in its path. Hint: the ball falls a distance equal to L to reach the lowest point in its motion.	Solution: (Section 6-7) The total kinetic + potential energy at point 1 equals the total kinetic + potential energy at point 2. Note: point 1 is the release point and point 2 is the lowest point in the ball's path. $PE_1 + KE_1 = PE_2 + KE_2$ $m\ g\ h_1 + \frac{1}{2}\ m\ v_1^2 = m\ g\ h_2 + \frac{1}{2}\ m\ v_2^2$ Note: the mass cancels where $h_1 = L$, $v_1 = 0$ m/s, let $h_2 = 0$ m, $v_2 = ?$ $g\ L + \frac{1}{2}\ (0\ m/s)^2 = g\ (0\ m) + \frac{1}{2}\ v_2^2$ $v_2^2 = 2\ g\ L$ and $v_2 = (2\ g\ L)^{1/2}$
Part b. Step 1. Use the law of conservation of energy speed at the top of the circular path above the peg.	The total kinetic + potential energy at point 1 equals the total kinetic + potential energy at point 3. Note: point 3 is the top of the circular path above the peg. Note point 3 equals the diameter of the circle that the ball follows. Since the peg is 0.8L below the support point, the radius of the circle is L - 0.80 L = 0.20 L. The diameter of the circle is 2(0.20 L) = 0.40 L. $PE_1 + KE_1 = PE_3 + KE_3$ $m\ g\ h_1 + \frac{1}{2}\ m\ v_1^2 = m\ g\ h_3 + \frac{1}{2}\ m\ v_3^2$ Note: the mass cancels where $h_1 = L$, $v_1 = 0$ m/s, $h_3 = 0.40$ L, $v_3 = ?$ $g\ L + \frac{1}{2}\ (0\ m/s)^2 = g\ (0.40\ L) + \frac{1}{2}\ v_3^2$ $\frac{1}{2}\ v_3^2 = g\ L - 0.40\ g\ L = 0.60\ g\ L$ $v_3 = (1.2\ g\ L)^{1/2}$